

On the Set of Betti Elements of a Puiseux monoid

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Abstract

Let M be a Puiseux monoid, that is, a monoid consisting of non-negative rationals (under standard addition). In this paper, we study factorizations in atomic Puiseux monoids through the lens of their associated Betti graphs. The Betti graph of $b \in M$ is the graph whose vertices are the factorizations of b with edges between factorizations that share at least one atom. If the Betti graph associated to b is disconnected, then we call b a Betti element of M . We explicitly compute the set of Betti elements for a large class of Puiseux monoids (the atomizations of certain infinite sequences of rationals). The process of atomization is quite useful to study the arithmetic of Puiseux monoids, and it has been actively considered in recent literature. This leads to an argument that for every positive integer n there exists an atomic Puiseux monoid with exactly n Betti elements.