

On orbit codes and lattices

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Abstract: Let $\lambda = (\lambda_1, \dots, \lambda_r)$ be some partition of a positive integer n , where $\lambda_1, \dots, \lambda_r$ are called as parts of λ . A finite abelian p -group (of rank r) corresponding to λ is given by,

$$G = \mathbb{Z}/p^{\lambda_1}\mathbb{Z} \oplus \dots \oplus \mathbb{Z}/p^{\lambda_r}\mathbb{Z},$$

where p is a prime number. From G , we design a variable length binary non-linear code, denoted by \mathcal{C}_λ , it is called as the *automorphism orbit code*. Each constituent in the decomposition of G corresponds to variable-length codewords which are referred as *automorphism orbit codewords*. The codewords of \mathcal{C}_λ are *automorphism orbit codewords*. The concept of *homomorphism codes* is rooted in the relationships between groups through homomorphisms, while the characterization of *automorphism orbit codes* involves a more intricate interplay of factors such as partitions of a number, orbits resulting from group action, as well as homomorphisms and automorphisms within groups. For some groups G and H , elements from $Hom(G, H)$ play a crucial role in establishing a foundational *cover* relation for the bit strings comprising the codewords of an *automorphism orbit code*. This mechanism serves as the basis for constructing a lattice framework housing variable-length non-linear codes.

Keywords: Finite abelian group, group action, homomorphism, automorphism, binary code.

Mathematics Subject Classification. 05E16, 05A17.

REFERENCES

- [1] Raja, R., Wagay, S. A.: Some invariants related to threshold and chain graphs, *Advances in Mathematics of Communications*. doi: 10.3934/amc.2023020 (2023).
- [3] Mesnager, S., Raja, R., Wagay, S. A.: On the computation of Seidel Laplacian eigenvalues for graph-based binary codes. *Discrete Mathematics*, 347 (2024) 113986
- [2] Mesnager, S., Raja, R.: Orbit codes of finite Abelian groups and lattices. *Discrete Mathematics* 347 (2024) 113900.

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