

The ideals of a numerical semigroup with embedding dimension two

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Abstract

Let S and Δ be numerical semigroups. We will say that S is an ideal of Δ if there exists $X \subseteq \Delta$ such that $S = (X + \Delta) \cup \{0\}$. In this work, we will study the ideals of a numerical semigroup of the form $\langle a, b \rangle$ with a and b positive integers such that $\gcd\{a, b\} = 1$. The main results that we have obtained are the following:

1. Given a numerical semigroup S and $\{a, b\} \subseteq \mathbb{N}$ such that $\gcd\{a, b\} = 1$, we present an algorithm that allows us to determine if S is an ideal of $\langle a, b \rangle$.
2. If S is a numerical semigroup, we show an algorithmic procedure to compute the set $\{\{a, b\} \subseteq \mathbb{N} \mid \gcd\{a, b\} = 1 \text{ and } S \text{ is an ideal of } \langle a, b \rangle\}$.
3. We obtain formulas to compute the multiplicity, Frobenius number and genus of the numerical semigroups of the form $(X + \langle a, b \rangle) \cup \{0\}$ in terms of X , a and b .

This is a joint work with J.C. Rosales.