ROOK POLYNOMIALS OF ALMOST SYMMETRIC ARF NUMERICAL SEMIGROUPS

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ABSTRACT

In this work, we give the rook polynomials of Young diagrams corresponding to almost symmetric Arf semigroups. Also, in this work, we give the rook polynomials of the complements of Young diagrams corresponding to almost symmetric Arf semigroups.

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Keywords Almost symmetric Arf semigroups, numerical sets, numerical semigroups, rook polynomials, Young diagrams.

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