ARITHMETIC VARIETIES OF NUMERICAL SEMIGROUPS

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A *submonoid* of $(\mathbb{N}, +)$ is a subset of $\mathbb{N}$ containing 0 that is closed under addition. A *numerical semigroup* is a submonoid S of $(\mathbb{N}, +)$ such that $\#(\mathbb{N} \setminus S) < \infty$, that is, $\mathbb{N} \setminus S$ has finite cardinality.

If S is a numerical semigroup, then $\mathrm{m}(S) = \min(S \setminus \{0\}), \mathrm{F}(S) = \max(\mathbb{Z} \setminus S)$ and $\mathrm{g}(S) = \#(\mathbb{N} \setminus S)$ are relevant invariants of S called *multiplicity*, *Frobenius number* and *genus of* S, respectively.

If A is a non-empty subset of $\mathbb{N}$, then we write $\langle A \rangle$ for the submonoid of $(\mathbb{N}, +)$ generated by A, that is,

$$\langle A \rangle = \{u_1 a_1 + \ldots + u_n a_n \mid n \in \mathbb{N} \setminus \{0\}, \{a_1, \ldots, a_n\} \subseteq A \text{ and } \{u_1, \ldots, u_n\} \subset \mathbb{N}\}.$$

It is well known that $\langle A \rangle$ is a numerical semigroup if and only if $\gcd(A) = 1$.

In this talk, we present the notion of arithmetic variety for numerical semigroups. We study various aspects related to these varieties such as the smallest arithmetic that contains a set of numerical semigroups and we exhibit the root three associated with an arithmetic variety. This tree is not locally finite; However, if the Frobenius number is fixed, the tree has many nodes and algorithms can be developed.

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