STRETCHED NUMERICAL SEMIGROUP AND ITS FROBENIUS NUMBER

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Let $H = \langle n_1, n_2, n_3, \ldots, n_e \rangle$ be a numerical semigroup with $n_1 < n_2 < \ldots < n_e$, generated minimally by e elements. For a field k, we denote
$$k[[H]] = k[[t^h \mid h \in H]]$$
be the semigroup ring of H over k and we write $\mathfrak{m}_H$ the maximal ideal of $k[[H]]$.

For an Artinian local ring $(A, \mathfrak{m})$, we say that A is **stretched** if $\mathfrak{m}^2$ is generated by a single element.

We call H is a **stretched** numerical semigroup if $k[[H]]/(f)$ is a stretched Artinian local ring for some $f \in \mathfrak{m}_H$.

In this talk we discuss about the Frobenius number $\mathrm{F}(H)$ of stretched numerical semigroup H particularly when $\mathrm{GCD}(n_1, n_2) = 1$.

Our results are as follows;

(1) If $\mathrm{GCD}(n_1, n_2) = 1$, then $\mathrm{F}(H) \leq \mathbb{F}_e(n_1, n_2) := n_1(n_2 - e + 1) - n_2$ and if $\mathrm{F}(H) = \mathbb{F}_e$, then H is stretched.

(2) If H is stretched, then $\mathrm{F}(H) \geq n_2(n_1 - e + 1) - n_1 = \mathbb{F}_e(n_1, n_2) - (e - 2)(n_2 - n_1)$.

(3) If H is stretched and if $e = 4$, then $\mathrm{GCD}(n_1, n_2) = 1$.

We will discuss several conjectures related to stretched numerical semigroups and almost symmetric numerical semigroups. In particular, we will determine all possibilities of $\mathrm{F}(H)$ for stretched almost symmetric semigroups in the case $e = 4$ for given (n_1, n_2).

This is a joint work with Kazufumi Eto, Naoyuki Matsuoka and Takahiro Numata.

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