SOME ASYMPTOTIC PROPERTIES OF SHIFTED NUMERICAL SEMIGROUPS

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Let $S = \langle n_0, n_1, \ldots, n_k \rangle$ be a numerical semigroup minimally generated by $n_0 < n_1 < \cdots < n_k$ and let $r_k = n_k - n_0$ be the width of S. For every non-negative integer n, consider the *shifted numerical semigroup* $S_n = \langle n_0 + n \cdot r_k, n_1 + n \cdot r_k, \ldots, n_k + n \cdot r_k \rangle$. When n is big enough, it is known that several properties of S_n do not depend on n; for instance, Vu proved in [3] that the Betti numbers of the defining ideal of S_n are always the same, proving a conjecture of Herzog and Srinivasan.

In this talk I will review some results on shifted numerical semigroups, and I will present new ones. In particular, I will focus on the notion of nearly Gorenstein numerical semigroup.

Nearly Gorenstein rings were introduced by Herzog, Hibi, and Stamate in 2019 for Cohen-Macaulay rings in [1], and they have been investigated in several contexts in the last few years. There is a clear analogous notion for numerical semigroups, which generalizes symmetric, pseudo-symmetric, and almost symmetric semigroups. In particular, in order to study shifted numerical semigroups, I will describe the concept of NG-vector introduced in [2], which characterizes the nearly Gorenstein property of S in terms of their minimal generators and pseudo-Frobenius numbers.

This is joint work with Dumitru Stamate.

References

[1] J. Herzog, T. Hibi, D.I. Stamate, *The trace of the canonical module*, Israel Journal of Mathematics **233** (2019), 133–165.

[2] A. Moscariello, F. Strazzanti, *Nearly Gorenstein vs almost Gorenstein affine monomial curves*, Mediterranean Journal of Mathematics **18** (2021), article number: 127.

[3] T. Vu, *Periodicity of Betti numbers of monomial curves*, Journal of Algebra **418** (2014), 66-90.