NUMERICAL SEMIGROUPS WITH QUASI MAXIMAL EMBEDDING DIMENSION

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Consider $S \subseteq \mathbb{N}$ a numerical semigroup. A subset A of S is a minimal system of generators of S if $S = \langle A \rangle = \{\sum_{i=1}^{n} \lambda_i a_i \mid a_i \in A, \lambda_i \in \mathbb{N}\}$, and $\langle B \rangle \neq S$ for any $B \subset A$. It is well known that every numerical semigroup has a unique minimal system of generators, denoted $\mathrm{msg}(S)$, which is a finite set ([1, Theorem 1.7]).

In this work, we will introduce a new class of numerical semigroups. Given $x \in \mathbb{N}$, the numerical semigroup S is a QMED(x)-semigroup if $S \setminus \{0\} = \{a + kx \mid a \in \mathrm{msg}(S) \text{ and } k \in \mathbb{N}\}$. Note that if x is the multiplicity of S then S is a Maximal Embedding Dimension numerical semigroup (for more information about MED-semigroups see [2] and [1]).

We show that the set of all QMED(x)-semigroups is a Frobenius pseudo-variety giving its tree configuration. Furthermore, we give formulas to obtain the Frobenius number, type, and genus of this class of semigroups.

References

[1] J. C. Rosales, P. A. García-Sánchez, *Numerical Semigroups*, Developments in Mathematics 20, Springer, New York, 2009.

[2] J.C. Rosales, P.A. García-Sánchez, J.I. García-García, M.B Branco, *Numerical semigroups with maximal embedding dimension*, Int. J. Commut. Rings **2** (2003) 47—53.

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